Adapting Venn diagrams for Non-Monotonic Reasoning

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Abstract. This paper introduces two systems that extend Venn diagrams: Venn_B and Venn_{BT} to incorporate theory of belief change and non-monotonicity in diagrammatic logic. Most traditional diagram systems adhere to monotonicity, with only a few exceptions. In typical diagram logical systems, if a diagram $\mathcal{D}$ is derivable from a set of diagrams $\triangle$, then $\mathcal{D}$ is also derivable from the set $\triangle \cup \mathcal{D}^{'}$, where $\mathcal{D}^{'}$ is any additional diagram. In contrast, Venn_B and Venn_{BT}, depart from monotonicity. For example, consider a belief base $\triangle$ containing the pieces of information such as 'there exist dogs', 'all dogs bark' and 'there exists Basenji and all Basenjis are dogs'. If we then learn that Basenjis are a species of barkless dogs (represented by diagram $\mathcal{D}_{'}$), these system prevent the derivation of 'all Basenji barks' (represented by diagram $\mathcal{D}$) from $\triangle \cup \mathcal{D}^{'}$, where $\mathcal{D}^{'}$. However, $\mathcal{D}$ was previously derivable from $\triangle$. Thus, Venn_B and Venn_{BT} exhibit non-monotonous properties by not allowing derivation from the expanded set. Instead, they incorporate belief change, adjusting the belief base to maintain consistency as the new information about barkless dogs contradicts the initial belief that all dogs possess the ability to bark.

Keywords: Non-monotonicity · Diagram logic · Belief revision · Typicality.

1 Abstract

In 1992, with the pioneering work of Sun-Joo [6], the field of Venn diagram logical systems started expanding, leading to the development of various subsequent systems [1,4,5]. However, all these systems adhere to monotonicity. That is, in these systems, if there is a set of diagrams, say $\triangle$, and a diagram $\mathcal{D}$ is derivable from $\triangle$ through some transformation rules [2] then $\mathcal{D}$ is also derivable from $\triangle \cup \mathcal{D}_{'}$ for any diagram $\mathcal{D}_{'}$. The only exception is found in Castro-Manzano [3], where nonmonotonic diagrammatic inference was showcased. However, this approach incorporate non-monotonicity by denying the combination of two contradictory diagrams, on the bases that these diagrams represent physical situations. But, in such a case an ideal situation appears where there is no contradiction can exists.

In this paper we take a novel approach by directly incorporating non-trivial contradictions into our syntactic framework. We propose a system where no diagram can arise from an contradictory diagram. However, two contradictory diagram can be combined. This approach mirrors a belief revision process, where in encountering contradiction upon obtaining new information prompts the revision of previously retained information to eliminate the contradiction. For instance, in both the systems Venn_B and Venn_{BT}, considering beliefs such as 'there exist dogs', 'all dogs bark', and 'there exist basenji, and all basenjis are dogs' these can be represented by the diagrams in Fig. 1, Fig. 2, and Fig. 3, respectively (where the circles 'Dog', 'Bark' and 'Basenji' denote sets of dogs, animals that bark, and the Basenji species, respectively).

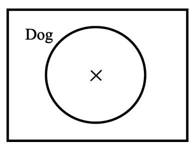

Fig. 1.

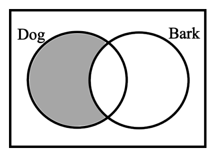

Fig. 2.

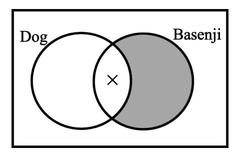

Fig. 3.

From these beliefs, we can deduce that 'all Basenjis bark' (Fig. 4). However, upon acquiring the new information that 'any existing Basenji is a bark-less dog' (Fig. 5), an inconsistent diagram emerges (Fig. 6) from the expanded belief base. Consequently, we adjust our earlier notion that 'all dogs bark' to eliminate this contradiction from our set of beliefs. Thus, we cannot anymore deduct 'all Basenjis bark' from our current expanded belief base.

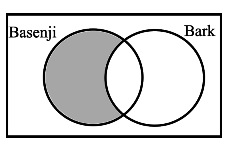

Fig. 4.

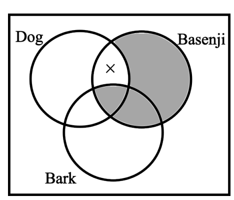

Fig. 5.

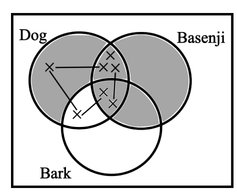

Fig. 6.

The system Venn_{BT} takes one step ahead of Venn_B by including defeasible reasoning along with belief revision. That is in Venn_{BT}, we can infer information based on our preference towards typically occurring cases along with revising our beliefs if we came across to contradictory information.. For example, if we have the information 'tweety is a bird' (Fig. 7), then we infer that 'tweety flies' (Fig. 8)[3]. This is due to the reason that we believe 'typically, bird flies' (Fig. 9)

[3] The circles 'Bird' and 'Fly' denotes the set of all birds and set of all things that flies respectively.

and even though we are aware about the non flying birds, like penguin, we show ignorance towards it. The dotted part of the circle 'Bird', denotes our ignorance. This is justified by taking the set cardinality of the dotted part less than the regular part, i.e. Bird $\cap$ Fly. Upon learning that Tweety is actually name for a penguin, we again face a contradictory situation and hence change our belief to 'Tweety does not fly' (Fig. 10). Thus, we cannot infer 'Tweety flies' anymore.

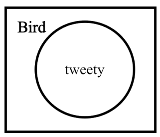

Fig. 7.

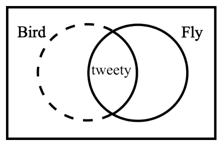

Fig. 8.

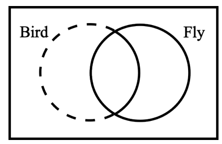

Fig. 9.

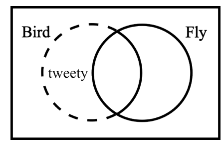

Fig. 10.

These systems thus not only shows non-monotonous behavior in Venn diagram but also combine two completely distinct areas, belief revision and diagrams. This work is currently under publication procedure.

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