Negative Impact of Drawing on Problem Solving: An Eye-Tracking Study on Non-linear Geometry Problems

Gudula Volbers[1], Janina Krawitz[2], Stanislaw Schukajlow[1], and Gilbert Greefrath[1]

[1] Universität Münster, Henriette-Son-Str. 19, 48149 Münster, Germany
[2] Universität Paderborn , Warburger Str. 100, 33098 Paderborn, Germany
gvolbers@.uni-muenster.de

Abstract. This study investigated the role of self-generated drawings in solving non-linear geometry problems. We aimed to identify factors for the repeatedly reported negative effects of making drawings on performance. Eighteen secondary students from upper secondary schools received either a drawing or drawing with highlighting instruction before solving the problems. Data were collected by recording the eye movements during task processing and a stimulated recall interview. Difficulties with the scale of the self-generated drawings and a lack of conceptual content knowledge explained the observed negative effects on students' performance.

Keywords: Diagram, Problem-Solving, Eye-tracking.

1 Theory and Research Questions

Making a drawing is considered a powerful strategy for solving problems, which helps in understanding the problem, finding appropriate solution procedures, and planning and monitoring the solution process [2,3]. Important factors for a positive effect of drawing are the quality of the drawing, the strategic knowledge about making and using a drawing in the problem-solving process, and the type of problem [3,4]. However, using the drawing strategy negatively affected performance in solving non-linear geometry problems, in which the area of similar figures had to be determined by a given scaling factor. For example: "The side of square C is four times as large as square D. If the area of square C is 1,440 cm2, what's the area of square D?" [2]. Two important reasons for the negative effect of self-generated drawings are as follows: First, the tendency of linear overgeneralizations triggered by drawing (i.e., applying linear models to non-linear situations). Second, the calculation of the area of the figure in the drawing, which does not correspond in scale with the measurements provided in the problem [1,2,6]. One possible explanation for the increase in linear overgeneralizations is that the students focused on linear activities by making a drawing [1,2]. However, an intervention that directed attention toward the area as the quantity to be calculated did not lead to an improvement in performance [2]. In this study, we analyzed the potential and obstacles to the use of self-generated drawings by linking students' eye movements and observed activities to the model of solving non-linear geometry problems. The model was developed based on Polya's problem-solving techniques and included the following phases: Understanding the problem, schema activation, devising a plan, carrying out the plan, validation, and documentation.

2 Method

The present study involved 18 students (10 female, mean age = 16.12 years) from upper secondary education, coming from two high-track schools and one comprehensive school. Students were randomly assigned to one of two groups: Before solving the problems, in one group students were asked to make a drawing and in the other group students were asked to make a drawing and to highlight the quantity that they should calculate. Both groups worked on a paper-and-pencil test. One problem was the task described in Section 1, and the other problem was an analog task for calculation the area of the circle. Data were collected by recording the eye movements during task processing using Tobii Pro Glasses 3 and a stimulated recall interview (SRI), in which students were asked to explain their solutions using the gaze replay videos. The gaze data videos were triangulated with the statements from the SRI. The data were analyzed by using a qualitative content analysis. The phases were analyzed according to the deductive categories from the model of solving non-linear geometry problems (see section 1). Eye movements were coded deductively-inductively based on a category system piloted in prior studies [5]. Categories of drawing use were: purely visual drawing use, visual-action drawing use (e.g. measuring a side length), and non-visual drawing use (e.g. using the drawing dimensions such as lengths in calculations).

3 Results and Discussion

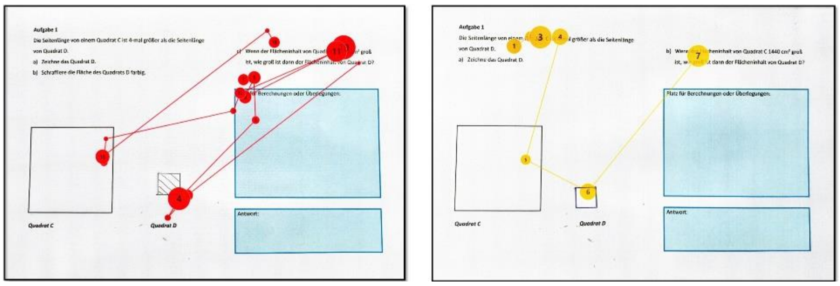

Fig. 1. Examples of eye movements

We found evidence for the use of drawings by all students. Drawings were used to understand the task, during the planning phase for remembering calculation methods (Fig. 1, left) or to validate the calculated result by estimating the size ratio of the two drawings (Fig. 1, right). During the execution phase of the solution, in 11 of 36 solutions, the drawings were used incorrectly to calculate the area of the smaller drawing. One student attempted to find the solution for the square task by using the drawing to pave the larger square with the smaller one, but she failed in transferring the paving strategy to the task of scaling circles. Concerning the linear overgeneralization we identified two types of "linear solvers". The first type applied linear overgeneralization without considering the problem (surface strategy) or due to a lack of alternative ideas on how to solve the problem In only one case, linear overgeneralization could surely be attributed to the activity of making a drawing. The second type first calculated the area of the small figure. After recognizing the incorrectness of this solution this type applied linear overgeneralization due to a lack of alternative approaches. This seems to be another important cause of the increase in linear overgeneralizations, rather than an attentional bias towards linear variables caused by drawing, as assumed in previous studies [1,2]. Students of both types sometimes had doubts about the correctness of the solution and tried to validate it by estimating the size ratio of the two drawings. However, some students only superficially checked that one figure was significantly larger than the other and did not recognize their mistake. Other students recognized the incorrectness of the solution but were not able to correct it. In addition to analyzing eye movements, the SRI revealed that some students perceived drawing as helpful for remembering the area formula for squares or circles, while others found it unhelpful because the drawings were not to scale. Some students found drawing helpful for certain problems (square problem) due to the paving strategy but not for others (circle problem). In summary, drawings were successfully used for remembering calculation methods or to validate the result. For many students, the discrepancy between the scale of the drawings and the given numerical values revealed to be a significant obstacle to integrating information from the diagram and text. This obstacle led to wrong solutions. The main reason for this difficulty appears to be a lack of conceptual knowledge about areas and similar figures, which seems to be a major factor contributing to the negative effect of drawing. Another reason is the lack of strategic knowledge about using a drawing during problem solving. These findings are in line with previous research results [1,3] and further extend prior studies by demonstrating that at least some students recognize their erroneous linear solutions but nonetheless adhere to them due to a lack of alternative approaches. Our findings have both theoretical implications for explaining the negative effect of drawing [1,2] and practical consequences concerning the suitability of drawing in solving non-linear geometry problems.

References

1. De Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2007). The Illusion of Linearity: From Analysis to Improvement. NY: Springer US
2. Krawitz, J., & Schukajlow, S. (2020). When can making a drawing hinder problem solving? Effect of the drawing strategy on linear overgeneralizations and problem solving. Frontiers in Psychology, 11(506). DOI: 10.3389/fpsyg.2020.00506
3. Rellensmann, J. (2019). Selbst erstellte Skizzen beim mathematischen Modellieren. Ergebnisse einer empirischen Untersuchung. Springer.
4. Pólya, G. (1945). How to Solve It: A New Aspect of Mathematical Method. Princeton University Press
5. Thomaneck, A., Vollstedt, M., & Schindler, M. (2022). What can eye movements tell about students' interpretations of contextual graphs? A methodological study on the use of the eye-mind hypothesis in the domain of functions. Frontiers in Education. 7. DOI: 10.3389/feduc.2022.1003740
6. Volbers, G., Schukajlow, S., Greefrath, G., & Krawitz, J. (2023). Zeichnen einer Skizze-(K)eine geeignete heuristische Strategie zur Lösung nicht-linearer Probleme? In IDMI-Primar Goethe-Universität Frankfurt (Hrsg.), Beiträge zum Mathematikunterricht 2022 (S. 231-234). WTM-Verlag