Meta-Analysis of Visualization Interventions in Mathematics Education

Johanna Schoenherr[1][0000-0002-6445-2917], Anselm R. Strohmaier[0000-0001-8339-9587], Stanislaw Schukajlow[3][0000-0003-0519-5276]

[1] Paderborn University, Warburger Str. 100, 33098 Paderborn, Germany
[2] Technical University of Munich, Arcisstr. 21, 80333, Munich, Germany
[3] University of Münster, Henriette-Son-Straße 19, 48149, Münster, Germany
johanna.schoenherr@uni-paderborn.de

Abstract. Visualization has a long tradition in mathematics education, and research on this topic has become more widespread in recent decades. In a meta-analysis (41 studies, N = 10,562), we aimed to synthesize the effects of learning with external visualizations on mathematics outcomes. We analyzed intervention (e.g., digital technology), learner (e.g., prior knowledge), and outcome characteristics (e.g., far transfer) as moderators. Overall, results of a random-effects model indicated a medium effect (g = .50, *CI* [0.379, 0.630]) of visualization interventions on mathematics learning, with significant heterogeneity. Moderator analyses revealed that effect sizes were higher in quasi-experimental studies and when compared with business-as-usual conditions. These results emphasize the effectiveness of external visualization as a powerful tool to support mathematics learning, with positive and lasting effects across age groups and mathematical topics.

Keywords: Visualization; Mathematics; Learning.

1 Introduction

External visualizations, which include graphs, shapes, physical objects, and statistical plots, are integral to mathematics learning and can be provided by educators or generated by learners. Visualization interventions in mathematics education describe a learning environment where students engage in activities designed to enhance learning through visualizations. When learners engage with external visualizations, bidirectional interactions between internal and external representations occur, serving various functions such as promoting conceptual understanding and aiding memory retention (Arcavi, 2003; Presmeg, 2006).

Previous reviews in science and mathematics education have reported positive effects of visualization interventions, with meta-analyses identifying benefits for specific learner groups and tools used for visualization (e.g., Sokolowski, 2018; Zhang et al., 2023). The overall effectiveness of visualizations in mathematics learning remains unexplored, and a comprehensive synthesis of their effects on learning outcomes is lacking, along with unclear conditions moderating the successful implementation of visualization interventions.

To understand the varied effects of visualization interventions in mathematics learning, we explore intervention, learner, and outcome characteristics. Intervention characteristics include study design, type of control conditions, visualization types, analog and digital media, and mathematical topics (e.g., geometry). Learner characteristics encompass education level and prior knowledge, and outcome characteristics include the type of outcome measure, near and far transfer, and immediate and delayed testing.

2 The present review

This meta-analysis aimed to assess the impact of visualization interventions on mathematics learning across all education levels. Research questions addressed the overall effect and potential moderating factors. Expected effect sizes were anticipated to align with medium effect sizes found in previous meta-analyses focusing specific groups of learners or tools (e.g., Sokolowski, 2018; Zhang et al., 2023).

3 Method

Following PRISMA guidelines, we systematically searched interdisciplinary databases (Web of Science, Scopus, and Education Resources Information Center) and highly ranked mathematics education journals. Search terms included four key visualization terms combined with relevant educational and mathematical terms. The search yielded 4,295 reports from 1990 to 2023. Applying inclusion and exclusion criteria—which were participants at any mathematics education level, focus on learning with external visualizations by employing a (quasi-)experimental design with a control group, reporting a math outcome measure and data for effect size calculation, and an available full text in English language—we identified 41 studies that were eligible for inclusion in this meta-analysis. We applied a qualitative content analysis to the full texts to systematically extract information on intervention characteristics, learner characteristics, outcome characteristics, and effect sizes. To extract effect sizes, the unbiased standardized mean difference (Hedges' g) was utilized, and three-level meta-analytic models were estimated. For testing moderators, separate meta-regressions were conducted with moderators as fixed effects.

4 Results

Our meta-analysis of 153 effect sizes with a total of 10,562 learners yielded an overall effect size of visualization interventions on cognitive outcomes in mathematics of $g = 0.50$. Meta-regression analyses revealed that quasi-experimental designs, business-as-usual control groups, and near transfer outcome measures were associated with larger effect sizes. Other moderators, including visualization type, analog or digital media used for visualization, and mathematical topic did not moderate the effects of visualization interventions on outcomes. There was no significant publication bias observed between peer-reviewed journal articles and non-reviewed grey literature.

5 Discussion

In this meta-analysis of 41 intervention studies conducted between 2010 and 2023, visualization interventions were found to have a medium overall effect (g = 0.50) on mathematics outcomes. This effect size aligns with previous meta-analyses (e.g., Zhang et al., 2023), providing robust support for the effectiveness of learning mathematics with visualizations across different contexts.

The analysis of study characteristics revealed that quasi-experimental designs and comparisons with business-as-usual control conditions were associated with larger effect sizes. Experimental studies with randomized designs and alternative treatment control conditions had slightly lower effect sizes, suggesting potential inflation in quasi-experimental studies. To obtain more reliable estimates, researchers are encouraged to employ randomized experimental designs.

Regarding intervention characteristics, no significant differences were found between various types of visualizations or mathematical topics. Both digital and analog visualizations were equally effective, challenging the assumption that technology necessarily enhances effectiveness. Further research is needed to explore how different technological tools influence the nature of mathematics learning.

The analysis of learner characteristics showed that visualization interventions benefited students across educational levels, with no significant differences between learners with low or average prior knowledge. However, individual learning characteristics, including prior knowledge, should be considered in designing effective visualization interventions.

Outcome characteristics indicated that visualization interventions supported diverse mathematics outcomes, including understanding, problem-solving, and problem posing. The positive effects extended to visual abilities, such as strategic knowledge about visualizations and mental rotation. Notably, the benefits persisted on near and far transfer posttests, as well as delayed posttests, suggesting sustainable promotion of mathematics learning beyond the intervention period by using visualizations.

Disclosure of Interests. The authors have no competing interests to declare that are relevant to the content of this article.

References

1. Arcavi, A.: The role of visual representations in the learning of mathematics. Educational Studies in Mathematics **52**(3), 215–241 (2003)
2. Presmeg, N. C.: Research on visualization in learning and teaching mathematics. In: Gutiérrez, A., Boero, P. (eds.) Handbook of research on the psychology of mathematics education, pp. 205–235. Sense Publishers, Rotterdam (2006)
3. Sokolowski, A.: The effects of using representations in elementary mathematics: Meta-analysis of research. IAFOR Journal of Education **6**(3), 129–152 (2018)
4. Zhang, Y., Wang, P., Jia, W., Zhang, A., Chen, G.: Dynamic visualization by GeoGebra for mathematics learning: a meta-analysis of 20 years of research. Journal of Research on Technology in Education, 1–22 (2023)